CENTRALE COMMISSIE VOORTENTAMEN WISKUNDE

Entrance Exam Wiskunde B

Date:	20 April 2026
Time:	13.30 – 16.30
Questions:	4

Please read the instructions below carefully before answering the questions. Failing to comply with these instructions may result in deduction of points.

Make sure your name is clearly written on every answer sheet.

Take a new answer sheet for every question.

Show all your calculations clearly. Illegible answers and answers without a calculation or an explanation of the use of your calculator are invalid.

Write your answers in ink. Do not use a pencil, except when drawing graphs. Do not use correction fluid.

You can use a basic scientific calculator. **Other equipment, like a graphing calculator, a calculator with the option of computing integrals, a formula chart, BINAS or a book with tables, is NOT permitted.**

On the last page of this exam you will find a list of formulas.

You can use a dictionary if it is approved by the invigilator.

Please **switch off your mobile telephone** and put it in your bag.

Points that can be scored for each item:

Question	1	2	3	4
a	6	5	6	3
b	6	5	3	7
c	5	4	5	6
d	3	6	6	
e	5			
Total	25	20	20	16

Grade $= \dfrac{\text{total points scored}}{9} + 1$

You will pass the exam if your grade is at least 5.5 .

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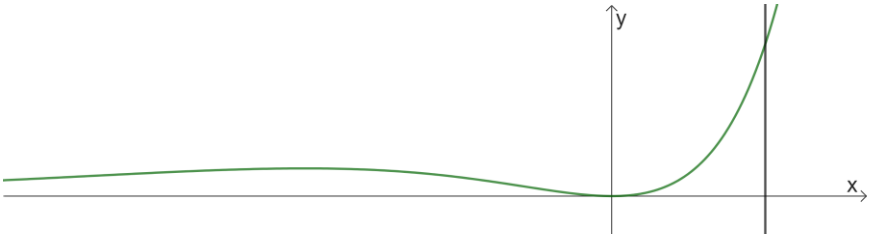
Question 1 – The square of an exponential function and a logarithmic function with its square

Take a new answer sheet for every question!

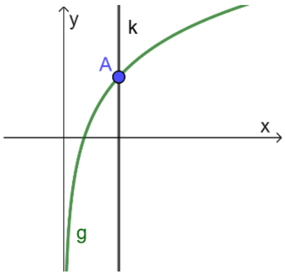
In the figure directly below, the graph of the function $f(x) = (\mathrm{e}^{2x} - \mathrm{e}^{x})^2$ and the vertical line $x = \ln\left(\sqrt{2}\right)$ are shown.



6pt **a** Compute exactly the maximal value of $f(x)$ for $x \leq 0$.

6pt **b** Compute exactly the area of the bounded region enclosed by the *x*-axis, the line $x = \ln\left(\sqrt{2}\right)$ and the graph of f.

In the figure on the right, the graph is shown of the function $g(x) = \ln(3x)$.
Line k is the vertical line with equation $x = \frac{1}{3}\mathrm{e}$.
Point A is the intersection of line k and the graph of g.



5pt **c** Compute exactly the coordinates of the intersection of the tangent line to the graph of g in point A and the *x*-axis.

The function $H(x) = x \cdot (\ln(3x))^2 - 2x \cdot \ln(3x) + 2x$ is an antiderivative of $h(x) = \left(g(x)\right)^2$.

3pt **d** Use an exact computation to show this.

V is the bounded region enclosed by the graph of g, the *x*-axis and line k.

5pt **e** Compute exactly the volume of the solid of revolution that is formed by rotating *V* around the *x*-axis.

Question 2 – A family of rational functions

Take a new answer sheet for every question!

For each real value of a, the function f_a is given by

$$f_a(x) = \frac{x^2 + x}{x + a}$$

There are two values of a for which the graph of f_a has a perforation

In some textbooks, this is called a removable discontinuity or a singularity.

5pt a Compute exactly the coordinates of this perforation for both values of a.

5pt b Compute exactly the values of a for which the graph of f_a has two extreme values.

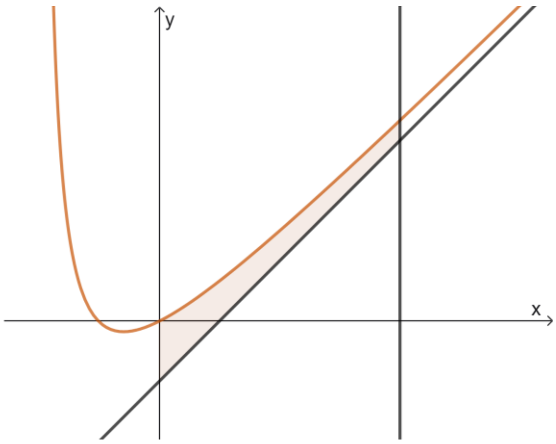
We now take $a = 2$. This yields the function

$$f_2(x) = \frac{x^2 + x}{x + 2}$$

4pt c Use an exact computation to find equations for the asymptotes of the graph of f_2.

V is the bounded region enclosed by the *y*-axis, the line with equation $y = x - 1$, the vertical line $x = 4$ and the graph of f_2, see the figure below.

6pt d Compute exactly the area of region *V*.

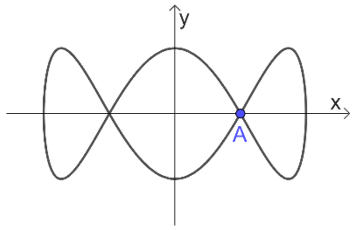


Question 3 – The path of a moving point

Take a new answer sheet for every question!

The movement of a point *P* is given by the parametric equations

$$\begin{cases} x(t) = 2\sin(t) \\ y(t) = \cos(3t) \end{cases} \quad \text{with } \ 0 \le t \le 2\pi.$$



In the figure on the right, the path of point *P* is shown.

6pt a Compute exactly the coordinates of the points on the path of *P* where the tangent line to this path is horizontal.

The path of point *P* intersects itself in point $A(1\,,0)$.

3pt b Use an exact computation to show this.

5pt c Compute algebraically the angle at which the path if *P* intersects itself in point A.

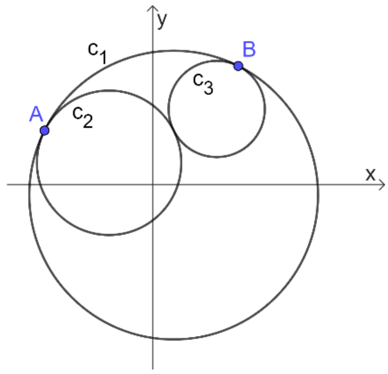
The line $y = \frac{1}{2}x$ intersects the path of *P* in several points.

6pt d Compute exactly the values of t in the interval $0 \le t \le 2\pi$ that correspond to these points.

Question 4 is on the next page

Question 4 – Touching circles

Take a new answer sheet for every question!



In the figure on the right, three circles are shown.

Circle c_1 has equation $(x-2)^2+(y+1)^2=180$.

Circle c_2 touches circle c_1 in point $A(-10,5)$ and passes through the centre M_1 of circle c_1.
From this, it follows that $M_2(-4,2)$ is the centre of circle c_2.

3pt a Use an exact computation to show this.

Circle c_3 touches circle c_1 in point $B(8,11)$.
The radius of circle c_3 is equal to $\sqrt{20}$.
In the figure, it appears that circle c_3 also touches circle c_2.

7pt b Investigate with an exact computation whether this is indeed the case.

Finally, a fourth circle c_4 is given (not in the figure).
Circle c_4 has centre $O(0,0)$ and circle c_4 touches the straight line through the points $A(-10,5)$ and $B(8,11)$.

6pt c Compute exactly an equation for circle c_4.

End of the exam.
*When you have finished the exam, check whether your **name** and the **question number** are on every answer sheet.*
Place the answer sheets in the correct order in the plastic folder and place the sheet with your data in the front in this folder.
*What should **not** be in the folder:* *- empty sheets, please leave them on your table;* *- sheets with only your name on it, please take them with you;* *- scrap paper;* *- these questions.*
This is the only way we can ensure a smooth correction of your exam work.
Remain seated until one of the invigilators collects your folder (or calls you).

Formula list wiskunde B

$\sin^2(x) + \cos^2(x) = 1$

$\sin(t + u) = \sin t \cos u + \cos t \sin u$

$\sin(t - u) = \sin t \cos u - \cos t \sin u$

$\cos(t + u) = \cos t \cos u - \sin t \sin u$

$\cos(t - u) = \cos t \cos u + \sin t \sin u$

$\sin(2t) = 2\sin(t)\cos(t)$

$\cos(2t) = \cos^2(t) - \sin^2(t) = 2\cos^2(t) - 1 = 1 - 2\sin^2(t)$

